Optimal Consolidation of Air Freight for an International Cargo Carrier

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Kıtalararası Kargo Taşıyıcısı için en iyi Havayolu Nakliye Birleşimi

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Özet

Havayolu kargo taşıyıcıları, elleçleme emeği ve aktarma maliyetlerinden tasarruf etmek amacıyla nakliyelerini birleştirirler. Bu çalışmada, uluslararası bir havayolu kargo taşıyıcısının birleştirme süreciyle ilgili planlama problemleri incelenmiştir. Operasyonel planlamada, uçuş ağı bilgisiyle birlikte terminal yer seçimlerini ve sığalarını göz önünde bulundurarak, en iyi birleştirme kararlarının havayolu kargo taşıyıcısı tarafından alınması beklenir. Bunun sonucunda, elde edilen en iyi birleştirme çözümü, aktarmayla ve birleştirilmiş kargo taşımacılığıyla sağlanan tasarrufun en büyüklenmesi hedeflenir. Birleştirme ve güzergâh atama problemi için set kaplama tipinde bir doğrusal programlama gösterimi oluşturulmuştur. Bu problem için kolon türetimi temelli bir çözüm yöntemi geliştirilmiştir. Sabit tutulan terminal ve uçuş ağı bilgileri göz önünde bulundurularak, terminal sığası artırımı problemi taktiksel düzeyde çalısılmıştır. Sığa artırım kararlarından dolayı, problemin gösterimi genişletilmiştir. Stratejik düzeyde yer seçim kararları

incelenmiştir. Bu problemin çözümü için, kolon türetimi yöntemini altyordam olarak kullanan sezgisel bir yöntem geliştirilmiştir. Geliştirilen yöntemlerin etkinlikleri, operasyonlarını çoğunlukla Avrupa'da gerçekleştiren bir uluslararası havayolu kargo taşıyıcısının gerçek verileri üzerinden incelenmiştir.

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Keywords: airfreight cargo; consolidation; linear programming; mixed integer programming ; column generation

Abstract

Air cargo carriers consolidate the freight in order to avoid extra handling effort and holding cost during transfers among the international hubs. We consider planning problems associated with the consolidation process of an international air cargo carrier. At the operational level of planning, the cargo carrier is concerned with optimal consolidation decisions given the locations and capacities of gateways with consolidation capability along with the flight network information. An optimal consolidation maximizes the savings due to transfer and transport of freight in a consolidated manner. We develop a set covering type linear programming problem formulation for this consolidation and routing problem; we also propose a column generation method to solve large-scale instances. At the tactical level, we study the expansion of gateway capacities keeping the gateway network and flight network as it is. The problem formulation is extended to cope with capacity expansion decisions

and the solution method is enhanced appropriately. At the strategical level, we consider decisions associated with selecting new locations for gateways. In order to solve this problem, the column generation method is employed as a subroutine in a heuristic algorithm. For our computational experiments, we use real-life data set from a European-based international air cargo carrier.

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Chapter 1

INTRODUCTION

Due to the vast scale of commerce, economic expansion of Asian markets and technological innovations, the globalization has become even more significant than it was in previous decades. Most of the commodity are being transported from one side to the other side of the world. As the customer satisfaction turns out to be a key factor, the shippers are more inclined to prefer air cargo for shorter and reliable delivery times. The air freight comprises of 43% of the global trade with a growth rate of 3.2% in tons per kilometer (Pearce, 2015). The lowest growth rate after the economic crisis falls above 2.6%, but it is expected to increase at a rate of 4% even in the worst case according to the forecasts.

Transportation of a shipment requires a series of operations to be accomplished such as picking up from the shipper, handling and sorting, storage, custom clearance, physical transportation and delivery to the customer. The integration among these operations becomes critical for a smooth door-to-door process; in this respect, a logistics provider bears the responsibility as an intermediary player to the shipper and the customer. As a result, the integrated management of in-house operations as well as efficient use of flight capacities over an effectively designed international hub network is crucial for an integrated intermediary player such as an air cargo carrier.

Study on passenger flight services is pretty common in the operations research literature, while the air cargo services remain quite unsolved (Feng et al., 2015).

Depending on the scale of the international hub network, the process of carrying a shipment from origin to destination comes with several planning issues that require

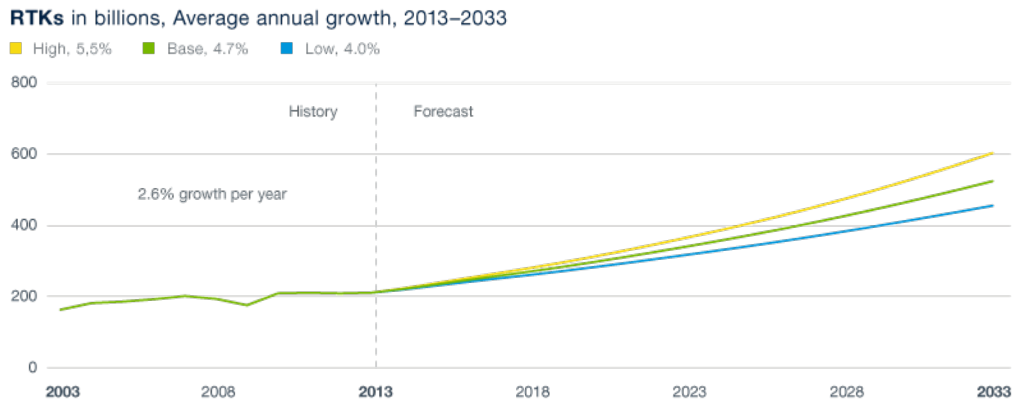

Figure 1.1: The annual growth rate of world air freight

decisions to be made on operational, tactical and strategical level. Firstly, the network structure along with the hubs is pretty complex by itself. A shipment does not necessarily follow the shortest path over the flight network from its origin to destination; it may be transferred from one flight to another a few times. Therefore among various flight routes, a shipment may follow longer paths with less handling due to transfers, while some others may follow shorter paths with more handling. Moreover, the shipments collected from different shippers might share some part of their routes, and therefore might be consolidated so that it is much easier to transfer the cargo from one flight to another. In addition to the selection of the route over the flight network, it is also important to determine where to consolidate the shipments during its route. Overall, even for a single shipment, route selection and consolidation decisions are very critical. Considering the entire flight network, these planning decisions get more and more complex.

For an air cargo carrier, decisions associated with consolidation plans are taken at all levels of a hierarchical planning process. The design of the hub network and selection of locations, where consolidation can be performed, is critical. Consolidation capacities of selected hubs affect the magnitude of savings due to consolidated transfers, while the route selection plays a key role in cost management.

In this study, we focus on efficient consolidation and routing of shipments to maximize the total savings obtained by efficient derouting and transportation of individual shipments. We develop a network representation of the problem, in which we embed consolidation of shipments and the resulting savings. To reduce the computational burden in solving the problem, we develop a set covering type mathematical formulation and use column generation.

Afterwards, we modify the problem formulation to integrate the capacity expansion decisions for the hubs with consolidation capability. As a result, we extend the set covering formulation. The capacity expansion problem can be solved in an iterative manner. First, we determine the profitable hubs to be invested in, and then we expand consolidation capacities of the selected hubs accordingly. Thereafter, we solve the extended set covering formulation to achieve effective consolidation and routing decisions by benefiting from the expanded consolidation capacities corresponding to the selected hubs.

Then, we further modify the problem formulation designed for the capacity expansion problem so that selected hubs may be opened up for consolidation operations. Considering the list of opened hubs that might be broadened at each iteration, we should simultaneously decide on how to allocate the limited consolidation capacity to those selected hubs. Consequently, we end up having the hub design decisions over the flight network and then new locations for the selected hubs equipped with dedicated consolidation capacity are attained by solving the gateway network design problem iteratively.

The remainder of the thesis is organized as follows. In Chapter 2, we describe the problem in detail, present a network representation and review the appropriate literature. In Chapter 3, we present the formulation of freight consolidation and routing problem and discuss a column generation algorithm and computational study. In Chapter 4, the formulation of gateway capacity expansion problem is presented along with a solution approach based on the column generation algorithm discussed in Chapter 3. We discuss the formulation of gateway network design problem in Chapter 5. We investigate the computational results for gateway network design problem in comparison to the gateway capacity expansion problem. Lastly, we conclude the study with several remarks, observations and future research opportunities in Chapter 6.

Chapter 2

PROBLEM DEFINITION AND NETWORK REPRESENTATION

An air freight forwarder collects the shipments from shippers at the dock, a station for unloading trucks, where the shipments are transmitted to the carrier (Amaruchkul and Lorchirachoonkul, 2011). Due to unavailability of direct flights from origin to destination over the network, the cargo carrier is in charge of the actual transportation process. The cargo carrier gets collected shipments and distributes them through several hub airports before being transferred to the destination hub (Feng et al., 2015).

This study focuses on air freight routing and consolidation operations of an air cargo carrier. As the individual shipments are transported from one hub to another, the cargo carrier aims at reducing the operational cost related to material handling and transportation rates (Li et al., 2009). The consolidation operation takes place to combine different shipments having different origins, destinations, or both in a container (Hall, 1987). By grouping the shipments, a cargo carrier intends to increase the container load resulting in a lower per unit transportation cost due to effective container utilization (Pan et al., 2013). Consolidating shipments also allows a more efficient handling of shipments and reduce operational costs. Accordingly, the cargo carrier wants to determine the best possible consolidation plan.

The demand of an origin-destination pair represents the total amount during the planning horizon, while the availability and amount of resources such as flight and consolidation capacity also covers the same horizon. In other words, we study a static version of the problem where both the demand and the availability of resources are aggregated. Therefore, we avoid using the time component and the availability of resources over time is neglected, even though we consider a finite planning horizon. Additionally, it is assumed that sufficient flight capacity is held at carrier's disposal to carry the individual or consolidated shipments during the planning horizon.

2.1 Problem Definition and Environment

For an international cargo carrier, the network is comprised of many origin, intermediary and destination locations. We describe an OD pair as transporting the shipments from the origin hub to the destination hub along a direct flight leg or a various connected flight legs.

A group of individual shipments collected from many suppliers are transported over the network; the shipments are loaded onto the flights as consolidated or individually. Typically, an OD route is composed of several flight legs connected via hubs at the end of each flight leg. At a destination hub denoted as a gateway, individual shipments are either sorted individually or consolidated together with other shipments in a container and routed to a new destination.

To perform consolidation, a gateway should be equipped with a number of containers. Each container has a finite volume. The consolidation capacity of a gateway indicates the total amount of individual shipments that can be assigned to all possible containers available at the gateway during the finite planning horizon.

Lee et al. (2006) argues that cargo scheduling and routing can be done efficiently in order to minimize waiting time and maximize resource. Therefore, air freight consolidation is a crucial issue in air cargo transportation planning. Feng et al. (2015) claim that the major reason of consolidating cargo is to make use of cutting down the expense of loading, unloading, sorting and inspection of commodities at each hub as well as the indirect transshipment cost resulting from the utilization level of the flight capacity. Consolidation ensures that a group of shipments put together into a container are treated as a single shipment. As a result, the shipments are transferred from one hub to another hub in a fast, cost saving and reliable fashion.

According to literature, there are 3 types of transportation consolidation strategies applied in practice; inventory, vehicle and terminal consolidations (Xu et al., 2006).

Inventory consolidation necessitates stocking items that might not be produced at the same time, waiting for the minimum amount of loading to be transported. Vehicle consolidation requires collecting the shipments together before routing the vehicle, while the consolidated items placed in the same load may be dropped off at different locations. Terminal consolidation is performed by accumulating the inventory level of items collected from different sources at the same location.

Vehicle consolidation corresponds to consolidation of similar items that may share part of their route from origin to destination. In the planning process of a vehicle consolidation operation, there are two essential co-related decisions; routing and consolidation. For each shipment, given a set of predetermined vehicle routes, an origin-destination route is determined as a sequence of vehicle routes; transfer from one vehicle to another along the route may be done individually for each shipment or together with other shipments, if they are all being transferred from the same vehicle. The problem we consider in this thesis corresponds to the vehicle consolidation problem.

In the context of air cargo transportation, a vehicle corresponds to a flight. An origin-destination path of a shipment is composed of a sequence of flights; it is called a flight route depicted over a flight network. For an OD pair, there could be alternative flight routes, which may differ from each other with respect to total transportation cost or distance and the number of transfers. Given a flight route for an OD pair, alternative consolidation schemes are possible depending on where shipments are consolidated and dismantled during transfers from on flight to another.

In essence, the least costly route also providing maximum savings due to consolidation can be selected individually for each shipment. However, it would require a network of unlimited flight capacities and gateways with infinite consolidation capacity. Therefore, routing and consolidation of shipments should be planned simultaneously. In this work, we assume that the flight capacities of the air cargo company aggregated over a finite planning horizon is more than sufficient when compared with total amount OD shipments over the same planning horizon period. Even though this assumption may sound restrictive for operational purposes, the main objective of this thesis is to solve a strategic planning problem.

2.2 Literature Review

The consolidation operations are comprehensively studied in the literature with respect to various modes of transportation. Liu et al. (2008) suggest a consolidation method for railways by designing a *blocking* network over the physical railroad network where blocks correspond to a combination of shipments of different OD pairs traveling concurrently. As the shipments transported from origin to destination, they may be subject to several blocking operations through regrouping the shipments at various railyard locations.

The same principle applies to the airline services as well. The shipments collected from multiple shippers possibly with different origins and destinations are consolidated and loaded onto the container. Consolidated shipments travel some part of their route together, while a shipment might be dismantled and reconsolidated a couple of times before arriving to the destination.

Although research on air cargo transportation is rare, we may still provide some studies addressing air cargo transportation planning problems. The air cargo transportation problems are studied under four major groups involved in the transportation process; shipper, forwarder, carrier and the owner of the distribution network (Feng et al., 2015). However, the research in literature essentially focuses on air forwarder, third party logistics carriers and a central distribution network owner as a decision maker.

Tyan et al. (2003) present a cargo consolidation problem as fulfilling service and capacity constraints. An integer programming model is formulated for the cargo consolidation problem. Optimal dispatch scheduling policies and assortment of alternative flight decisions are taken by $3PL$ provider. Additionally, loose and skid shipments are preferred for enhanced utilization of flight capacity at the cargo terminals. Tyan et al. (2003) investigate the inventory consolidation through collecting the outbound shipments from multiple manufacturers at the dock, where the shipments are kept to be loaded into flight with respect to the volatile demand and space constraints of flight capacity. Then, the consolidated shipment is routed to serve $B2B$ and $B2C$ customer stops subsequently. Huang and Chi (2007) examine the air freight consolidation problem, which is formulated as a mixed integer programming model. An air freight forwarder plans the time of outbound shipments received from shippers. Assignment of shipment to different flights is another important decision taken by air freight forwarders, since the segmentation of flights differs due to possible routing options and type of fleet. Due to the complexity of air cargo structure rate, the volume and weight of shipments have to be considered before loading shipments into the container (Huang and Chi, 2007). Then those consolidated shipments might be assigned to the following leg until the desirable flight leg would be realized. Additionally, the possible routing selection leads the consolidation problem to a well known set covering problem environment. Feasible consolidated shipments are treated as a set to convert the problem into the set covering model. To solve the relevant set covering model, a Lagrangian relaxation approach is applied. Li et al. (2009) present a freight consolidation problem with respect to capacitated container number. The resulting model turns to be a mixed integer linear programming model. Li et al. (2009) examine the air freight forwarder responsibility for cargo loading planning problem as well, while introducing the reserved available capacity for the assignment of shipments to flights with distinct transportation rates. As a result, a fixed number of containers is dictated on the cargo loading problem. The freight consolidation problem is solved by proposing a variant of neighborhood search heuristic.

Irnich (2000) and Li et al. (2012) focus on the cargo loading planning problem. Irnich (2000) surveys the assignment of shipments to a vehicle, while $3PL$ provider should also consider the type of vehicle capacity, speed of vehicle, earliest pick-up date and latest delivery time of assignments. A mixed integer programming model is formulated for the cargo loading problem. To solve the mixed integer programming model, optimization based heuristic is proposed by enumerating the possible route and vehicle combinations. Li et al. (2012) investigate a shipment planning problem to build a decision model for the unsplittable freight consolidation problem. The corresponding problem is formulated as a mixed integer linear programming model. Air freight forwarders are allowed to combine minor loads from shippers into larger packages. Li et al. (2012) study an unsplittable shipment consolidation problem where the commodities of an OD pair should be traveled over a single route from origin to destination. Li et al. (2012) contemplate the potential arrangement of shipments sending by different origin air cargo terminals with the intention of taking efficient routing decisions. Both Lagrangian relaxation and local branching techniques are utilized to solve the shipment planning problem.

Shi et al. (2012) investigate an integration optimization problem to satisfy the demand over built-to-order supply chain. An integration optimization problem is solved by an integer programming model considering the trade-off between distribution cost and lead time dependent demand satisfaction. Therefore, the objective of the study is to maximize total savings acquired by delivery of demand through the network. In (Shi et al., 2012), a central decision maker takes the responsibility for opening decision and the design of distribution centers. Moreover, the decision maker should contemplate a model that satisfies both target service level and demand level of retailers. Shi et al. (2012) propose a Lagrangian relaxation and subgradient based optimization algorithm to attack corresponding integer programming model as a solution procedure.

2.3 An Illustrative Example

In order to exemplify both routing and consolidation process of a set of OD pairs consisting of 12 hubs, we provide a representation of this example in Figure 2.1. On this network, a hub is denoted by a node and a flight is denoted by a directed arc.

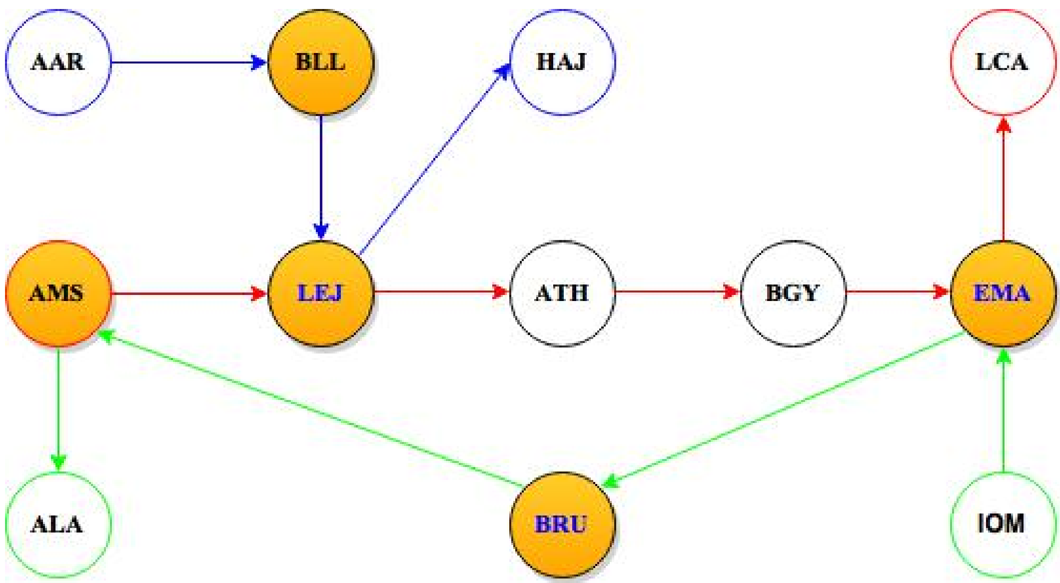

Figure 2.1: An illustration of the flight route network

Over this network, there are three distinct OD pairs: $AAR - HAJ$, $AMS - LCA$ and $IOM - ALA$. For each OD pair, the flight routes can be prescribed as follows, respectively;

$$\begin{aligned}&\{AAR - BLL - LEJ - HAJ\}\,,\\&\{AMS - LEJ - ATH - BGY - EMA - LCA\}\,,\\&\{IOM - EMA - BRU - AMS - ALA\}\,.\end{aligned}$$

The origin-destination nodes of an OD pair are circled with the same color; a path with directed arcs of the same color corresponds to the flight route of the OD pair. Orange nodes represent the gateways, where consolidation can be performed. Nodes with blue font text represent the hubs where the last consolidation operation takes place for the corresponding OD pair.

For the OD pair $AAR - HAJ$, the shipment is transported from the origin node ARR and visits BLL and LEJ to be consolidated, then it is transported to HAJ as consolidated and dismantled at the sink node HAJ.

For the OD pair $AMS - LCA$, the shipments may not be traveled together along the flight route. While some of shipments might be consolidated at AMS and traveled to LEJ as consolidated, the remaining shipments might be transferred to LEJ as unconsolidated. Then, they might share the same path for the rest of flight route as dismantled at ATH and transferred to EMA where the final consolidation is performed.

The flight route for the OD pair $IOM - ALA$ begins with the origin node IOM as unconsolidated. At node EMA, partial demand of the OD pair is consolidated and transferred to BRU; where the final consolidation is performed. Subsequently, the consolidated items are transferred from BRU to the sink node ALA as consolidated.

According to Figure 2.1, different flight routes coinciding with the same orange nodes might cause the ambiguity in consolidation operations performed. Considering the case where both of the flight routes for the OD pairs $AAR - HAJ$ and $AMS - LCA$ coincide with LEJ, where the available consolidation capacity might fall short of the amount to fully consolidate the demands for both of the OD pairs. For this reason, we should take into consideration alternative consolidation schemes that cannot be exemplified through the flight route network representation.

In order to avoid the ambiguity in consolidation operations and lack of information on the flight route network, we may develop a representation of the flight route, which includes more details regarding the locations of consolidation. For this purpose, we describe the sub-route network to exploit where consolidation operations take place on each flight route.

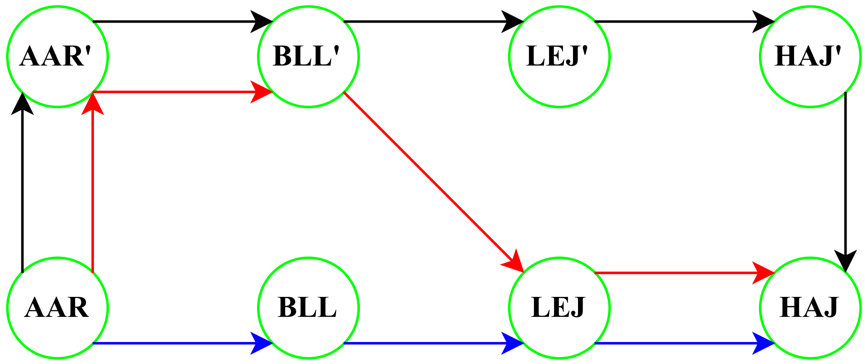

Figure 2.2: An illustration of the sub-route network

For a given flight route network, we create copies of the original nodes at a second layer as illustrated in Figure 2.2 for the OD pair of $AAR - HAJ$. Copies of nodes (denoted by ') indicate the consolidation operations performed at the preceding node on the corresponding flight route. We assume that shipments are not consolidated at the sink node. An upward arc, which can only be represented at the origin node, and an arc pointing from the original node to its successor second layer copy represent the consolidation performed at the corresponding gateway; while a downward arc, which can only be represented at the second layer copy of the sink node, and an arc pointing from the second layer copy to its successor original node represent dismantling. An arc from a second layer node to another second layer node shows that shipments are transported as consolidated. In Figure 2.2, three alternative sub-routes are exemplified:

- The first sub-route shown by black arcs visits AAR'; then, it follows the second layer of nodes until the destination node HAJ. On this sub-route, the consolidation is performed at AAR from where the consolidated shipments are transferred to destination HAJ as consolidated.
- On the second sub-route shown with red arcs, shipments are consolidated at origin AAR and then transferred to BLL as consolidated. Then, they are

dismantled at BLL and are transported until destination HAJ as deconsolidated.

- The third sub-route shown with blue arcs follows the deconsolidated path; shipments are never consolidated over this sub-route.

The savings due to consolidation are obtained by avoiding excess handling while shipments are transferred from one gateway to another. Using arc cost coefficients on the second layer arcs, we may easily embed the saving information into the sub-route network.

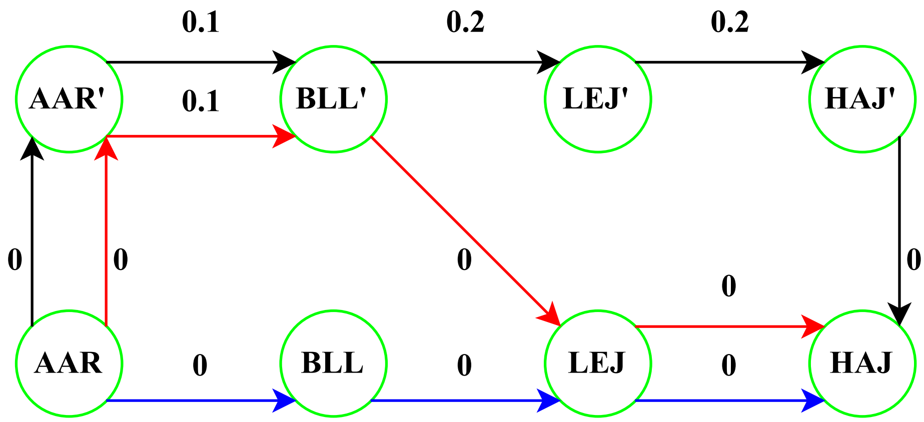

Figure 2.3: An illustration savings structure embedded in the sub-route network

In Figure 2.3, we show the saving coefficients for the sub-route network example in Figure 2.2. We take the rate of transporting shipments following original deconsolidated path as 1.0, whereas the rate of transporting shipments along second layer arcs is set to be 0.9. Once the shipments are consolidated at a second layer node, successive flight to another second layer node causes a reduction in the transportation rate as taken 0.8 in our example.

For the first sub-route shown with black arcs, the shipments are consolidated at the origin AAR and transferred to BLL as consolidated with a unit saving of 0.1. Due to traveling along successive second layer arcs, the unit savings are set to 0.2 for both of the flight legs $BLL - LEJ$ and $LEJ - HAJ$. The savings of flight legs; AAR' to BLL', BLL' to LEJ' and LEJ' to HAJ' are realized as 0.1, 0.2 and 0.2, respectively. Therefore, it contributes 0.5 to the overall profit of associated sub-route.

We also assume that a shipment traveling as consolidated uses the gateway consolidation capacity of the gateways corresponding to the tail node of an arc, i.e. dismantling operations do not require any gateway capacity. For the second subroute shown with red arcs, the shipments use the consolidation capacity of origin AAR and are dismantled at BLL, from which they travel along the original layer arcs. Since no other consolidation operation is performed over this flight route; none of the consolidation capacities of BLL, LEJ and HAJ is used.

2.4 Methodology

We characterize three planning problems that are associated with each other within a hierarchical planning framework: freight consolidation and routing, gateway capacity expansion and gateway network design problems from bottom to top in the planning hierarchy.

In Chapter 3, we propose a linear programming problem formulation that determines the consolidation and routing of shipments by assigning OD pairs to pre-determined sub-routes.

In Chapter 4, the problem is enriched with additional decisions on expanding the gateway capacities. This problem considers the allocation of a limited number of extra containers to bump the consolidation capacities of selected gateways. For this purpose, we propose a mixed integer linear programming problem formulation. That is an extension of the one in Chapter 3.

In Chapter 5, a network design problem is studied to determine critical gateways over the flight network. This problem also contemplates the allocation of limited number of extra containers to the selected gateways with no consolidation capacity beforehand. We modify the mixed integer linear programming problem formulation presented in Chapter 4, in such a way that new hubs can be selected to be equipped with consolidation capability.

Chapter 3

FREIGHT CONSOLIDATION AND ROUTING PROBLEM

In freight consolidation and routing plans, the carrier is concerned with transshipping the freight from their origins to destinations over the flight network with an optimal routing and consolidation plan while maximizing the total savings due to consolidation and respecting consolidation capacities of gateways. To solve this problem, we develop a mathematical model in the form of a linear programming problem formulation; it turns out to be a set-covering type formulation. Due to the scale of the problem, we use the column generation method as an alternative to enumeration of all decision variables apriori. We test the computational performance of our method with real-life data from an international carrier.

3.1 Mathematical Formulation

Given the set of all shipments, we may generate the information of all OD pairs by aggregating the shipments with identical OD information on different dates. We also assume that the flight route information is available for any possible OD pair, and the sub-routes for each flight network may be given or generated by enumeration. As a result, the sub-route information along with the corresponding savings are known along with other necessary information on the flight network.

I	set of OD pairs
J	set of all sub-routes (potential consolidation routes)
J_i	set of all sub-routes of OD pair i
G	set of gateways
d_i	shipment amount of OD pair i
c_{ij}	consolidation saving of shipment of OD pair i routed via sub-route j, $j \in J_i$
N_g	consolidation capacity of gateway g (in number of containers)
C	container capacity

Table 3.1: Notation for the mathematical formulation

In Table 3.1, we introduce the notation for the problem. The decision variable defined as

$$y_{ij} : \text{consolidation percentage of shipment of } OD \text{ pair } i \text{ routed via sub-route } j.$$

The resulting mathematical formulation of the routing and consolidation problem is as follows:

$$\max \sum_{i \in I} \sum_{j \in J_i} c_{ij} d_i y_{ij} \tag{3.1}$$

s.t.

$$\sum_{j \in J_i} y_{ij} \leq 1, \qquad \forall i \in I, \tag{3.2}$$

$$\sum_{i \in I} \sum_{j \in J_i : g \in j^1} d_i y_{ij} \leq N_g C, \qquad \forall g \in G, \tag{3.3}$$

$$y_{ij} \geq 0, \qquad \forall i \in I, j \in J_i. \tag{3.4}$$

The objective function is to maximize the total savings obtained by transporting shipments as consolidated. Constraint (3.2) ensures for an OD pair that partial assignment to different sub-tours cannot exceed 1 (i.e. 100%); we refer to this constraint as the assignment constraint. Constraint (3.3) ensures that the amount of consolidation at a gateway is limited by its gateway capacity. We refer to constraint (3.3) as the capacity constraint. Constraint (3.4) is the non-negativity of the decision variable.

The mathematical model presented above provides a solution to linear programming problem with respect to given a set of upper bounds enforced on gateway consolidation capacities. The sub-routes passing over a certain gateway might exploit partial fraction of its capacity. As a result, the gateway capacities on the network are consumed by assignment of (several) OD pairs to sub-routes passing through the associated gateways.

We may enhance the mathematical model by introducing a new constraint that enables to justify the consolidation amount at each gateway by enforcing a lower bound on the amount of consolidation. The lower bound constraint can be written as;[2]

$$\max \sum_{i\in I}\sum_{j\in J_i} c_{ij} d_i y_{ij} \tag{3.5}$$

s.t.

$$\sum_{j\in J_i} y_{ij} \leq 1, \qquad \forall i \in I, \tag{3.6}$$

$$\sum_{i\in I}\sum_{j\in J_i : g\in j} d_i y_{ij} \leq N_g C, \qquad \forall g \in G, \tag{3.7}$$

$$\sum_{i\in I}\sum_{j\in J_i : g\in j} d_i y_{ij} \geq L_g \qquad \forall g \in G, \tag{3.8}$$

$$y_{ij} \geq 0, \qquad \forall i \in I, j \in J_i. \tag{3.9}$$

where L_g is the lower bound on the amount of consolidation capacity for the gateway g.

Constraint (3.8) ensures that sum of consolidated amount of OD pairs routed via gateway g is greater than the minimum justifiable consolidation amount enforced for the corresponding gateway. Since under-utilization of a gateway is prohibited by inserting the lower bound constraint (3.8), it is likely that used gateways are better utilized.

3.2 Solution Approach: A Column Generation Algorithm

Solving the linear programming model (3.1)-(3.4) requires generating all possible sub-routes in advance. Depending on the size of the flight network, number of hubs

[2] buraya bakalim

and number of distinct OD pairs, this may result in a very large scale problem due to the number of decision variables.

In essence, generating all possible sub-routes might be computationally impossible for large networks. In this respect, column generation is a common tool to resort to for problem formulations of set-covering type.

The computational complexity of this problem formulation stems from number of decision variables equivalent to all possible sub-route combinations. For a flight route of n intermediate hub stops, we may generate 2^{n-1} different sub-routes for an OD pair. For example, the data we use in our computational study consists of 35905 OD pairs and corresponds to 488560 possible sub-routes. Consequently, we have 488560 many decision variables. In order to generate the problem including all possible decision variables, one should first generate the information of the sub-routes for each flight route. However, the pre-processing of generating all possible sub-routes is computationally inefficient due to the size of the flight network and number of OD pairs. In addition to that, most of the variables will be equal to zero in an optimal solution.

Column generation is a convenient decomposition algorithm used to solve large-scale linear programming problems by separating the original problem into a restricted master problem and a pricing sub-problem (Dantzig and Wolfe, 1960). The restricted master problem is restricted with respect to the set of decision variables (corresponding to the columns of the primal problem); it does not include all the possible sub-routes for all OD pairs. In essence, for each OD pair, it contains only a subset of the feasible sub-routes. Column generation is an iterative algorithm, which adds new columns (decision variables) to the restricted master problem at every iteration until an optimal solution to the restricted master problem is proven to be also optimal for the original master problem with all possible columns. An initial feasible solution of a restricted master problem is required to begin with the idea lying behind the column generation method derives from dispensing with non-basic variables in the optimal solution, so that only the basic variables or even a subset of them are employed for inserting a new column into the basis (Barnhart et al., 1996). At this stage, a pricing sub-problem is solved to generate new column(s) violating the dual feasibility. From one to another consecutive iteration, a non-basic column is entered in the basis to constitute the optimal basis of final iteration (Bazaraa et al., 2004). The iterations should be followed until the dual feasibility is attained.

Revisiting the linear programming problem formulation (3.1)-(3.4), we observe that there are ($|I|$x$|J|$) many columns and ($|I|$+$|G|$) many rows in the original master problem. However, to generate the restricted master problem, we only need $|I|$ columns and $|I|$+$|G|$ rows to initialize.

In the column generation method, the optimality is reached by adding new columns that violate the dual feasibility. For this purpose, we check whether there are any columns that have to enter the basis. For this purpose, we first develop the formulation for the dual problem of (3.1)-(3.4). In the dual formulation, variables t_i correspond to the dual variables associated with constraints (3.2) and variables w_g correspond to the dual variables associated with constraints (3.3). Hence, t_i explains the dual variable of utilization of selecting sub-route j on the OD pair i and w_g denotes the dual variable of reserved capacity at each gateway g. The dual problem is formulated as follows:

$$\min \sum_{i \in I} t_i + \sum_{g \in G} N_g C w_g \tag{3.10}$$

s.t.

$$t_i + \sum_{g \in j} d_i w_g \geq c_{ij} d_i, \qquad \forall i \in I, j \in J_i, \tag{3.11}$$

$$t_i \geq 0, \qquad \forall i \in I, \tag{3.12}$$

$$w_g \geq 0, \qquad \forall g \in G. \tag{3.13}$$

Based on the dual formulation of the problem, we can derive the reduced cost function to indicate the rate of improvement of the objective function coefficient on the relating decision variable in advance of the decision variable becomes cost-effective. According to the dual problem formulation, the reduced cost function is given as

$$c_{ij} d_i - t_i - \sum_{g \in j} d_i w_g \qquad \forall i \in I, j \in J_i.$$

3.2.1 Pricing Sub-Problem

Given a primal solution and the corresponding dual values, the pricing sub-problem finds out the promising columns to enter the basis of the restricted master problem.

After solving the restricted master problem at some iterations, we obtain the dual prices pertaining to each constraint. Based on this information, we check whether there is a positive reduced cost column to enter the basis. In order to achieve this check implicitly instead of evaluating all possible columns, the pricing sub-problem calculates the maximum violation of a non-existing column with respect to feasibility of constraint (3.11). Accordingly, the pricing problem becomes;

$$\max_{i \in I, \bar{J}_i \in J} \{c_{ij} d_i - t_i - \sum_{g \in j} d_i w_g\} \tag{3.14}$$

where $\bar{J}_i \subset J_i$ contains the sub-routes that are not in the restricted master problem yet; $w_g \geq 0$ and $t_i \geq 0$ are values of the dual variables.

Note that the objective function (3.14) corresponds the reduced coefficient of non-basic variables. A new column is added to the restricted master problem only if the objective function value is as it signals the violation of dual problem constraint (3.11). For all sub-routes generated throughout the OD pair set, we find a column with a maximum non-negative reduced cost and we add corresponding column to the restricted master problem.

To solve the problem formulation (3.14), we use a network representation inspired by the sub-route consolidation network in Figure 2.2. A pricing sub-problem network is provided in Figure 3.1; where we aim at maximizing the reduced cost for all possible sub-routes of each OD pair located in the non-basic variable set by solving a longest path problem.

As illustrated in Figure 3.1 , we create a two-layer network where each layer of nodes corresponds to the hubs that the OD pair visits. The second layer nodes represents the consolidation operations performed at the corresponding node with the exception of the destination node. An upward arc and an arc pointing from the original node to its successor second layer copy denote the consolidation performed at the corresponding gateway; whereas a downward arc and an arc pointing from the second layer copy to its successor original node represent dismantling. An arc from a second layer node to another second layer node denotes the consolidated shipment.

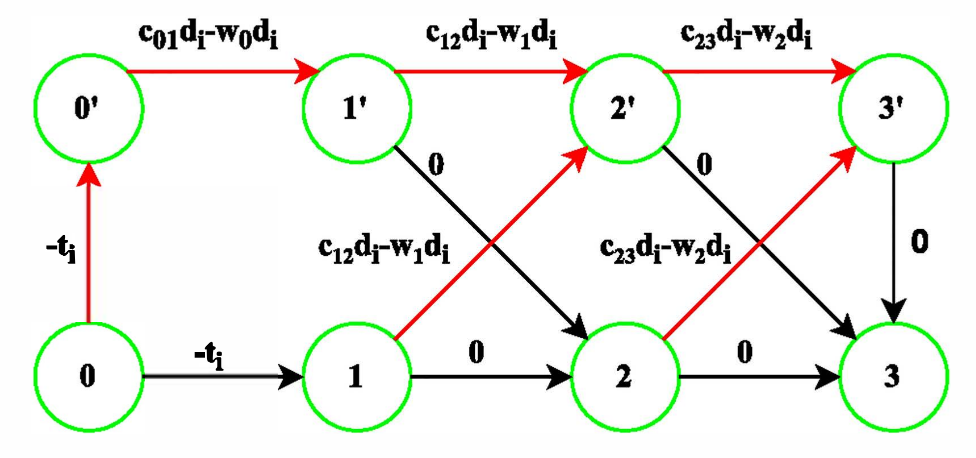

Figure 3.1: An illustration of the pricing sub-problem network for OD pair i with origin at node 0 and destination at node 3

The distance of a node is defined by the longest path from the origin. Accordingly, the distance of the destination corresponds to the longest path for a given sub-route of an OD pair. The longest path problem belongs to the class of NP-hard problems, it can still be solvable in polynomial time by a topological sorting algorithm regarding to the assumption of acyclic directed graph (Schrijver, 2003).

A modified version of topological sorting algorithm is developed to find out the distances for each node on the pricing sub-problem. Accordingly, we calculate the reduced cost of pricing sub-problem. The topological sorting algorithm exploits the acyclic property of directed network. For any graph $G = (N, A)$ linked with nodes $u \in N$, $v \in N$ and arc pairs $(u, v) \in A$, a topological ordering can be constructed as $u<v$, if node v is admissible from node u (Ahuja et al., 1993). Topological sorting algorithm is applicable if and only if there is no negative cycle on the network and the arcs should be directed. Runtime complexity of topological ordering algorithm is achieved by $O(|N^2|+|A|)$ (Zhou and Mueller, 2003).

The original topological sorting algorithm is designed for the shortest path problems. However it can be adapted to the longest path problem in case the graph is directed and acyclic.

In the very first step of the modified topological sorting algorithm in Algorithm 1, the node distances are initialized as zero for the origin node and $-\infty$ for any other node on the network. Then, the algorithm visits each node to identify admissible arcs from visited nodes to successor node. While visiting a node, the algorithm updates the distance of successor nodes based on the distance label of the tail node and the cost of arc from the tail to the successor node. Hence, the algorithm iterates as a distance label algorithm; in each iteration, a distance label is updated when the sum of the distance label of the tail node and the arc cost from the tail to the successor node is strictly greater than the distance label of the corresponding node. The algorithm has to be iterated until the following conditions hold that any node distance on the network cannot be updated and we represent the longest path at the destination node as the termination criteria.

We provide the topological sorting algorithm in the following form.

Algorithm 1: Modified Topological Sorting Algorithm

Let $G = \{N, A\}$, $u \in N$, $v \in N$ and $(u, v) \in A$

Set $dist[origin] = 0$, all remainder nodes $dist[v] = -\infty$

Create $pred[v] = \varnothing$ representing predecessor of v in the longest path from source node to v

forall v such that $(u, v) \in A$ & $pred[destination] = \varnothing$

    if $(dist[v] < dist[u] + weight(u, v))$ **do**

        $dist[v] \leftarrow dist[u] + weight(u, v)$

        $pred[v] \leftarrow u$

    end if

end forall

3.2.2 Column Generation Algorithm

We devise a column generation algorithm for the freight consolidation and routing problem as presented Algorithm 2. The algorithm proceeds with line (3) conditioned to search at least a single column having positive reduced cost among the non-basic variable set. In order to iterate lines (3-15), we need to solve the restricted master problem to obtain dual variables of utilization of selecting a sub-route and reserved consolidation capacity. Consequently, we implement a topological sorting algorithm to find out the longest paths for each OD pair. We solve the pricing sub-problem; so that a positive reduced cost column for corresponding OD pair is added to restricted master problem. After all, we iterate lines (6-8), in which we explore the column having maximum reduced cost out of the promising columns we have found by solving pricing sub-problem. And then, we add the promising column to the restricted master problem. At the final part of the algorithm, we solve the restricted master problem to check whether there is any column having positive reduced cost. If so, we need to iterate lines (3-15) again until obtaining the dual feasibility.

Algorithm 2: Algorithm for the Freight Consolidation and Routing Problem

1 Initialize RMP for the Freight Consolidation and Routing Problem
2 Set $found = true$
3 **while** $found$ **do**
4   solve RMP
5   obtain dual variables of t_i and w_g
6   **forall** $i \in I$
7   solve the pricing sub-problem for finding the longest path
8   **end forall**
9   set $found = false$ if no column with positive reduced cost is found
10   **if** $found = true$
11   **forall** $i \in I$
12   add the column to the RMP
13   **end forall**
14   **end if**
15 **end while**

3.3 Computational Results

Several computational studies are conducted related to the freight consolidation and routing problem. We compare two solution approaches to deal with the corresponding problem; generating all sub-routes and solving the original master problem, and column generation algorithm. The problems are solved using real life demand data provided by an international air cargo carrier. We use several values between 10000 and 50000 units as gateway capacities, as the gateway capacities are known to be in that range. The computational study in this section is based on the aggregated demand data of OD pairs over the network.

The data set consists of 488560 sub-routes generated from 33731 different OD pairs. The aggregated demands of OD pair are also provided in the data set. The implementations are done using CPLEX Studio 12.6 and Microsoft Visual Studio 2012 running on Intel i7, 3.2 gigahertz 64-bit operating system with 24 gigabyte RAM in order to solve optimization problems.

In Table 3.2, we study the freight consolidation and routing problem with respect to different minimum justifiable consolidation amounts. The very first column consisting of UB, LB 10%, LB 20% and LB 30% refers to solving the freight consolidation and routing problem with upper bound constraint (3.3) and combining upper bound with lower bound constraint (3.8) where lower bounds are given as the percentage of upper bound values. OFV represents the objective value of total savings obtained from the consolidated shipments. The column denoted as Limited/Unlimited stands for the objective function gap with respect to the optimal solution of unlimited consolidation capacity; i.e. consolidation capacity is equal to ∞. Variable column splits into two parts indicating the number of variables that are equal to 1 (taking integer values) and the number of variables that are taking values in $(0, 1)$ (taking fractional values). Variables with integer values denote that the shipment of an OD pair is transferred via a single sub-route, whereas the shipments are distributed partially to multiple sub-routes in case the variable takes fractional values. Following two columns denoted as Full and Partial refer to whether the gateway consolidation capacity is fully or partially used. In the Table 3.2, we omit the gateways, where the consolidation is not performed. Binding % denotes the percentage of fully consolidated gateways out of the total number of gateways disregarding the ones where any consolidation operation is not performed. All solutions in Table 3.2 are optimal solutions.

	N_gC	OFV	Limited/Unlimited	Time (in secs)	Variable (I/F)	Full	Partial	Binding %
UB	10000	168195	96.2%	1019.5	20059/98	88	178	33.1%
	20000	268481	93.9%	1006.4	23060/81	68	198	25.6%
	30000	352449	92.0%	1005.8	24490/76	60	206	22.6%
	40000	425731	90.3%	1000.9	25648/69	53	213	19.9%
	50000	492617	88.8%	999.1	26808/56	46	220	17.3%
LB 10%	10000	165127	96.2%	1003.3	15675/97	88	101	46.6%
	20000	259892	94.1%	1002.1	15512/75	68	93	42.2%
	30000	336515	92.3%	999.3	14807/71	60	82	42.2%
	40000	404830	90.7%	996.5	14712/60	53	78	40.4%
	50000	463454	89.5%	996.5	14712/57	46	73	38.7%
LB 20%	10000	159717	96.4%	999.9	12243/94	88	73	54.7%
	20000	248400	94.3%	996.9	11056/74	68	63	51.9%
	30000	318144	92.8%	994.6	10410/72	60	52	53.6%
	40000	381615	91.3%	992.5	9935/60	53	47	53.0%
	50000	432504	90.2%	992.1	10753/55	46	42	52.3%
LB 30%	10000	152995	96.5%	999.7	9499/91	88	54	61.9%
	20000	234875	94.7%	998.1	8217/75	68	44	60.7%
	30000	303968	93.1%	994.2	7966/71	60	35	63.2%
	40000	365458	91.7%	993.3	8788/61	53	34	60.9%
	50000	409440	90.7%	992.9	8588/50	46	29	61.3%

Table 3.2: Results for the freight consolidation and routing problem with respect to different consolidation capacity and lower bound on the consolidation capacity

As can be seen in Figure 3.2, imposing a lower bound on the justifiable consolidation capacity does not affect the behavior of shipment consolidation pattern significantly. While the justifiable consolidation amount rises, some of the gateways are not used for consolidation. However, the critical gateways (i.e. gateways that are fully used) do not change. Accordingly, the objective function value decreases when we impose higher lower bound on the justifiable consolidation amount.

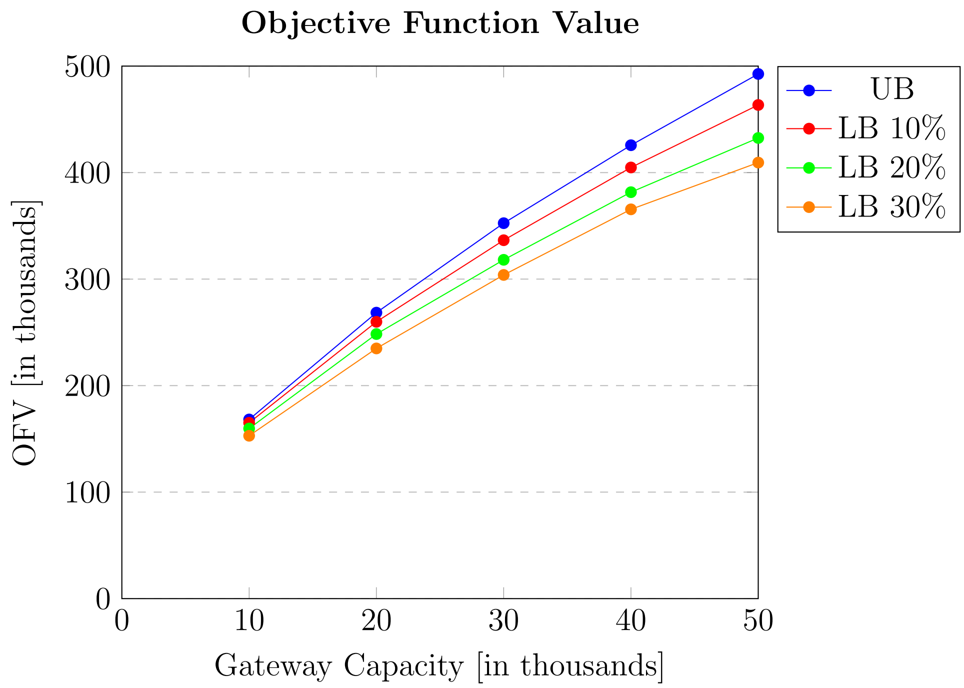

Figure 3.2: The objective function value results for the freight consolidation and routing problem with respect to different minimum justifiable consolidation amounts

In Figure 3.3, we also observe that some of the gateways preserves their consolidation operations regardless of the justifiable consolidation amount due to high usage of those gateways. Therefore, the percentage of binding gateways rises as the lower bound on the justifiable consolidation amount increases. Another important aspect of Table 3.2 is that the number of variables with fractional values remain almost constant regardless of different lower bound constraints. Additionally, a rise in the consolidation capacity of gateways results in using a single sub-route over an OD pair as expected.

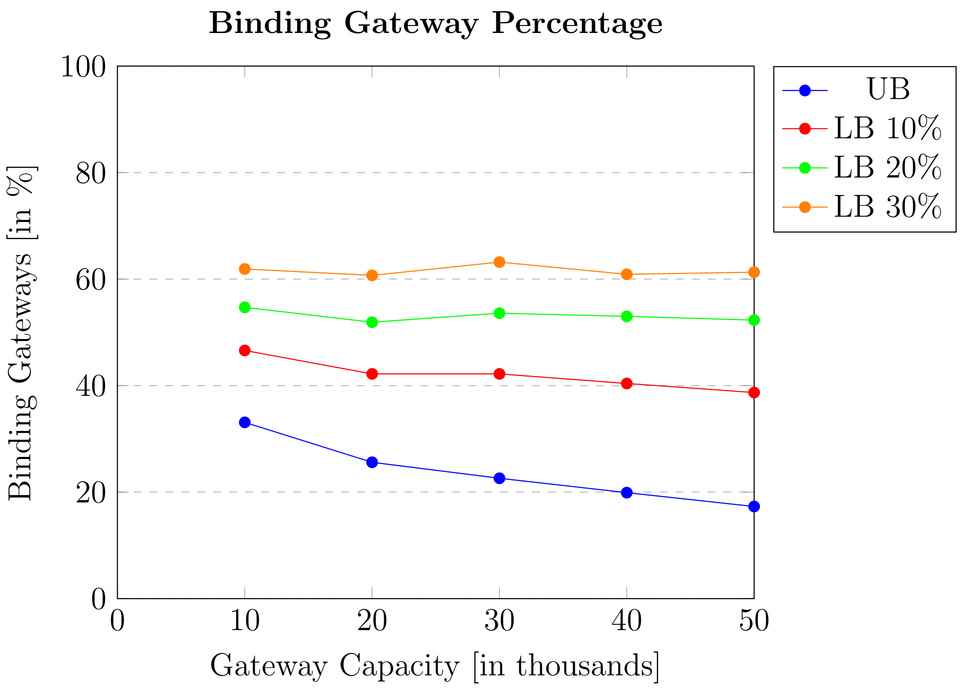

Figure 3.3: The binding gateway percentage results for the freight consolidation and routing problem with respect to different minimum justifiable consolidation amounts

In Table 3.3, we examine the freight consolidation and routing problem by comparing column generation algorithm with linear programming problem solution. The columns of Time[LP_{UB}], Time[$LP_{LB10\%}$], Time[$LP_{LB20\%}$], Time[$LP_{LB30\%}$] and Time[CG] stand for the required computational time of linear programming models for different minimum justifiable consolidation amount and column generation method to solve the the freight consolidation and routing problem, whereas Number of Columns represents the required additional number of columns to be added in the restricted master problem.

N_gC	Time[LP_{UB}]	Time[$LP_{LB10\%}$]	Time[$LP_{LB20\%}$]	Time[$LP_{LB30\%}$]	Time[CG]	Number of Columns
10000	1019.5	1003.3	999.9	999.7	109.7	89791
20000	1006.4	1002.1	996.9	998.1	108.9	97815
30000	1005.8	999.3	994.6	994.2	109.4	98434
40000	1000.9	996.5	992.5	993.3	108.2	101639
50000	999.1	996.5	992.1	992.9	108.5	87205

Table 3.3: Results for the freight consolidation and routing problem with respect to different solution procedures

Due to the long pre-processing operation for generating sub-routes, we expect to cut down the excess computational time by using column generation. As expected, column generation approach yields 9 times faster results with regards to linear programming solution, since it takes almost 13 minutes to generate all possible sub-routes on the network.

Chapter 4

GATEWAY CAPACITY EXPANSION PROBLEM

In gateway capacity expansion problem, we consider a network consisting of gateways with pre-determined consolidation capacities while there are containers available to be assigned to gateways. The scope of the gateway capacity expansion problem is to find out the most profitable hubs to raise their consolidation capacity and to determine the potential amount of capacity expansion. As a result, some of the gateway capacities might be expanded as a tactical decision process.

To solve the capacity expansion problem, we extend the set-covering type formulation developed for consolidation and routing problem by modifying the upper bound constraint with a decision variable representing the additional container capacity. First, we need to introduce the parameters of the gateway capacity expansion problem.

N : Available number of containers for consolidation

K : Consolidation capacity allocated for each container

We also introduce a decision variable as

z_g : number of additional containers assigned to gateway g

By adding a new decision variable z_g, the linear programming model turns into a mixed integer linear programming model. Since we neglect the cost of assigning extra containers to hubs, we do not modify the objective function. The resulting $MILP$ formulation is given as follows:

$$\max \sum_{i \in I} \sum_{j \in J_i} c_{ij} d_i y_{ij} \tag{4.1}$$

s.t.

$$\sum_{j \in J_i} y_{ij} \leq 1, \qquad \forall i \in I, \tag{4.2}$$

$$\sum_{i \in I} \sum_{j \in J_i : g \in j} d_i y_{ij} \leq C + K z_g, \qquad \forall g \in G, \tag{4.3}$$

$$\sum_{g \in G} z_g \leq N, \tag{4.4}$$

$$y_{ij} \geq 0, \qquad \forall i \in I, j \in J_i, \tag{4.5}$$

$$z_g \in \mathbb{Z}^+, \qquad \forall g \in G. \tag{4.6}$$

Constraint (4.3) ensures that the summation of consolidated amount of shipments is restricted by extra added container capacity in addition to the original gateway capacity. Constraint (4.4) ensures that we only add a limited number of containers, which represents the number of containers on hand. New decision variables are restricted to non-negative integer values, constraint (4.6).

In general, network design problems necessitate the solution procedures of combinatorial optimization including dynamic programming, dual ascent procedures, optimization-based heuristics and integer programming decomposition approaches (Ahuja et al., 1993). We solve the $MILP$ formulation of the capacity expansion problem in two approaches. The first approach is an iterative algorithm, that opens new gateways at each iteration. The second approach relies on solving LP relaxation of the problem and using rounding in order to satisfy integrality constraints.

4.1 An Iterative Algorithm

In the first stage of the iterative approach, we set apart the extra capacities and solve the original formulation with respect to given gateway capacities. Then, we expand the consolidation capacity of the profitable gateways and solve the new problem with column generation at each iteration in order to find a sufficiently good solution for the gateway capacity expansion problem.

The approach we developed for the gateway capacity expansion problem is presented in Algorithm 3. In addition to the freight consolidation and routing problem, we introduce several parameters into the algorithm. N represents the available container number, *bestcolumnsize* represents the desirable number of columns to be added at each iteration, *weightsortedOD* is a vector to sort OD pairs with respect to OD pair demands in non-increasing order; while *weightsortedgateway* is a vector to sort gateways with respect to corresponding dual variable of w_g in non-increasing order.

The algorithm starts with generating a vector to sort the OD pair demands in non-increasing order, since we observe that the gateway consolidation decisions are taken according to the critical OD pairs having relatively high demand. At the next step, we iterate lines (9-26) to find a sufficient solution for the capacity expansion problem with respect to pre-determined gateway capacities. During the iteration; depending on the OD pair demand information, we solve the pricing sub-problem to obtain the reduced cost of longest paths. Then, we sort the positive dual variables w_g in non-increasing order; the most profitable gateways are identified. After that, we iterate lines (20-25) to assign the available containers to corresponding gateways. We iteratively solve the algorithm until no available container is left.

The straightforward way of thinking for the gateway capacity expansion is based on finding a single gateway to raise its capacity at each iteration, so that resulting solution promises the optimal solution with respect to the number of containers and container capacity. However, it would be computationally expensive. Therefore, we observe that the information related to shadow price of reserved gateway capacity indicates us the most profitable gateways to be allocated extra capacity. At line (20) of the algorithm, we iteratively expand the gateway capacities until all available containers are assigned to the gateways. Then, we solve the updated restricted master problem again in order to take decision on the routing and consolidation options with respect to the extra capacities inserted into the network.

Algorithm 3: Algorithm for the Gateway Capacity Expansion Problem

Initialize RMP for the Gateway Capacity Expansion Problem

Set $found = true$

$weightsortedOD = \{\}$ / Sorting OD pairs w.r.t. demand

$weightsortedgateway = \{\}$ / Sorting gateways w.r.t. w_g

$bestcolumnsize = 5000$ / Desirable # of columns to be added

forall $i \in I$

    generate $weightsortedOD$ / In non-increasing order

end forall

while $found$ **do**

    obtain dual variables of t_i and w_g

    forall $weightsortedOD$ & $bestcolumnsize$

        solve the pricing sub-problem for finding the longest path

        add the column to the RMP

    end forall

    solve updated RMP

    set $found = false$ if no column with positive reduced cost is found

    forall $g \in G$

        generate $weightsortedgateway$ / In non-increasing order

    end forall

    while $N! = 0$ & $weightsortedgateway! = 0$ & $found$ **do**

        forall N & $weightsortedgateway$

            add container capacity to the corresponding column

            set $N = N - 1$;

        end forall

    end while

end while

4.2 A Relaxation Based Algorithm

Due to the computational effort required for the iterative algorithm and the quality of the solutions in Section 4.1, we next develop a heuristic based approach. In the heuristic approach, we remove the integrality constraints (4.6) and replace them by non-negativity constraints. We let u be the dual variable associated with constraint (4.4). The dual of the problem (4.1) - (4.6) is given as

$$\min \sum_{i \in I} t_i + \sum_{g \in G} w_g C + uN \tag{4.7}$$

s.t.

$$t_i + \sum_{g \in j} d_i w_g \geq c_{ij} d_i, \qquad \forall i \in I, j \in J_i, \tag{4.8}$$

$$u - K w_g \geq 0, \qquad \forall g \in G, \tag{4.9}$$

$$t_i \geq 0, \qquad \forall i \in I, \tag{4.10}$$

$$w_g \geq 0, \qquad \forall g \in G, \tag{4.11}$$

$$u \geq 0. \tag{4.12}$$

The algorithm that we use to solve the gateway capacity expansion problem is presented Algorithm 4. We start by relaxing $MILP$ formulation and initializing restricted master problem. To iterate *while* loop, we first have to solve the restricted master problem to obtain corresponding dual variables. Then, we use a topological algorithm to determine the longest paths for each OD pair. The pricing sub-problem is solved according to the longest path information and we get the reduced cost corresponding to the longest path of OD pairs. Then, we add the column(s) that have to enter the restricted master problem and we solve the updated restricted master problem to check whether there is left any column having positive reduced cost. The algorithm iterates until we reach the dual feasibility. At the end of the algorithm, we solve $MILP$ formulation corresponding to RMP obtained at the end of column generation.

By solving the gateway capacity expansion problem by column generation, we obtain an optimal solution to the LP relaxation of the problem (4.1) - (4.6). However, the solution obtained by this approach does not necessarily satisfies the integrality constraints (4.6). For this reason, after obtaining an optimal LP relaxation, we add

Algorithm 4: Algorithm for the Gateway Network Design Problem

Obtain LP relaxation of (4.1) - (4.6)
Initialize RMP for the Gateway Capacity Expansion Problem
Set $found = true$
while $found$ **do**
solve RMP
    obtain dual variables of u, t_i and w_g
    forall $i \in I$
        solve the pricing sub-problem for finding the longest path
        add the column with maximum positive reduced cost to the RMP
    end forall
    set $found = false$ if no column with positive reduced cost is found
end while Solve $MILP$ corresponding to RMP

integrality constraints for variables z_g and solve the $MILP$ problem formulation to optimality. The approach does not guarantee optimality. However, in our experiments, it performed better than the iterative approach in both computational time and quality of solutions.

4.3 Computational Results

In the following part, we present a computational study related to the gateway capacity expansion problem. Due to the mixed integer linear programming formulation, we obtain the optimal solution of capacity expansion problem through an integer programming solver. A proposed column generation algorithm solves the capacity expansion problem iteratively. While solving the resulting column generation algorithm, we realize that the performance of column generation majorly depends on skillful selection of entering column(s) during the pricing sub-problem process.

Therefore, we utilize the shadow price information of dual variable w_g by sorting in non-increasing order. Then, we set a constant on the number of columns to be added at each pricing sub-problem iteration instead of directly adding all of those columns having positive reduced cost. This approach allows us to reach a good quality solution easily even though it takes longer to reach an optimal solution. For this reason we stop column generation when the improvement between two successive column generation iterations is less than 1.0%. We illustrate this in Table 4.1, where we study two different column selection criteria to solve an iteration of the gateway capacity expansion problem. CG stands for adding one column for each OD pair having positive reduced cost to the restricted master problem, whereas $Best_{5000}$ stands for adding 5000 columns to the restricted master problem at each pricing subproblem iteration. Additionally, those best 5000 columns are selected among the columns having positive reduced cost by considering $weightsortedgateway$. It[CG] and It[$Best_{5000}$] represent the required number of iterations to reach the termination, i.e. all promising columns are already added to the restricted master problem. We observe that the total number of columns added to the restricted master problem significantly reduces by $Best_{5000}$ selection criterion, even though computation time is slightly longer.

C	It[CG]	It[$Best_{5000}$]	Time[CG]	Time[$Best_{5000}$]	Nb Columns[CG]	Nb Columns[$Best_{5000}$]
10000	5	11	103.7	172.7	89791	36718
50000	5	11	102.7	161.4	87205	41973

Table 4.1: Results for the gateway capacity expansion problem with respect to different column selection criteria

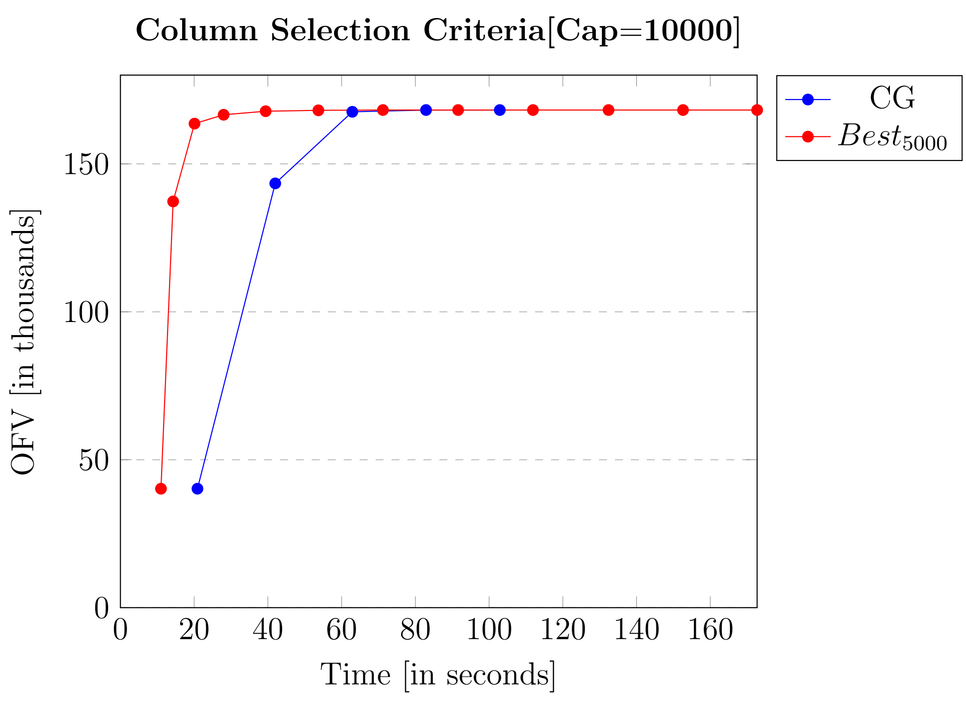

Figure 4.1: The objective function value results for gateway capacity expansion problem with respect to different column selection criteria

In Figure 4.1, we see that adding all promising columns at each pricing sub-problem yields better results in terms of computation time. However, $Best_{5000}$ selection criterion reaches good quality solutions in a few steps.

In the following discussion, we present the results obtained by computational experiments on the gateway capacity expansion problem with respect to different solution procedures in Table 4.2. First three columns refers to initial capacity of gateways, container capacity as to be assigned an extra capacity and number of containers available in the model, respectively. OFV[IP] stands for the optimal solution of the problem; whereas OFV[BC] refers to adding fixed number of columns at each iteration (at most 5000). At the end of column generation, we add gateway capacities to gateways for which the corresponding dual variable is positive. OFV[One] refers to assigning one container at each iteration (and solving column generation to optimality). OFV[CER] stands for the solutions obtained by relaxation heuristic. OFV[BC], OFV[One] and OFV[CER] are solved through utilizing column generation method. On the other hand, Time[IP], Time[BC], Time[One] and Time[CER] represent the required computational time (in seconds) for respective solution procedures.

In Table 4.2, we conduct several computational experiments using the parameters of gateway capacity, container capacity and number of available containers where IP solutions correspond to the optimal solution of resulting problem cases. Assigning container capacities one by one to gateways is computationally expensive, it promises near optimal solution. When we consider adding capacities to a set of promising gateways, the computation time is reduced drastically and results in almost 10 times faster computationally inexpensive solutions against one by one gateway capacity expansion method. However, the quality of solutions decreases. Relaxation heuristic is the best approach in terms of computation time. In our experiments, it solves each instance to optimality, even though this is not guaranteed.

C	K	N	OFV[IP]	OFV[BC]	OFV[One]	OFV[CER]	Time[IP]	Time[BC]	Time[One]	Time[CER]
10000	1000	100	196762	186436	196700	196762	993.7	208.8	2466.8	131.7
		1000	378356	301559	371521	378356	985.7	507.4	24241.9	66.4
	10000	10	196762	193724	196469	196762	991.7	134.1	277.5	131.7
		100	378356	327055	362676	378356	987.1	203.7	2468.7	66.8
30000	1000	100	382449	369369	382449	382449	984.7	199.2	2454.8	109.7
		1000	582619	486987	581690	582619	989.7	620.0	24412.7	66.3
	10000	10	382449	378096	382449	382449	989.3	124.4	268.2	109.2
		100	582619	513062	575551	582619	988.7	194.0	2452.6	67.5
50000	1000	100	522617	509584	522617	522617	983.5	169.9	2426.9	109.8
		1000	735650	632499	735650	735650	987.8	692.0	24328.6	69.1
	10000	10	522617	518596	522617	522617	987.7	96.5	240.6	108.8
		100	735650	656000	732187	735650	988.2	166.9	2436.1	67.9

Table 4.2: Results for the gateway capacity expansion problem with respect to different solution procedures

We also present optimality gaps for different approaches in Table 4.3. The performance of the column generation algorithm employing adding capacities to a set of promising gateways fluctuates depending on different model parameters. Generally, it performs well under limited number of available container instances; whereas the gap become wider as we have a plenty of available containers.

C	K	N	GAP[BC]	GAP[One]	GAP[CER]
10000	1000	100	5.25%	0.03%	0%
		1000	20.30%	1.81%	0%
	10000	10	1.54%	0.15%	0%
		100	13.56%	4.14%	0%
30000	1000	100	3.42%	0.00%	0%
		1000	16.41%	0.16%	0%
	10000	10	1.14%	0.00%	0%
		100	11.94%	1.21%	0%
50000	1000	100	2.49%	0.00%	0%
		1000	14.02%	0.00%	0%
	10000	10	0.77%	0.00%	0%
		100	10.83%	0.47%	0%

Table 4.3: Optimality gap for different solution procedures with respect to the IP solution

Chapter 5

GATEWAY NETWORK DESIGN PROBLEM

In gateway network design problem, the location of gateways and the OD pair demands are known in advance. However, the gateways are not capable of consolidation operation due to lack of consolidation capacity. Skillful assignment of containers to the gateways is the primary objective of gateway network design problem. Therefore, the gateways that are assigned container capacity can perform consolidation operation.

Since the initial design of the network does not contain any gateway with the consolidation capability, we seek for those, which are located critically on the flight route network. Accompanied by offering limited consolidation capacity, i.e. containers for the sake of our context, the solution of gateway network design problem plays an important role in maintaining maximum possible saving through the use of limited capacity. After identifying the critical gateways, we need to allocate the desirable amount of consolidation capacity by utilizing the sub-route information.

To solve the gateway network design problem, we use the $MILP$ formulation (4.1) - (4.6) by setting $C = 0$. In the modified constraint, we do not have any initial capacities but use capacity expansion variables to assign consolidation capacity to gateways. Thus, the container capacity K with a multiplier of associated positive integer decision variable z_g will be the only consolidation capacity provided for gateways.

Solution Approach and Computational Study

To solve the gateway network design problem, we use the relaxation based heuristic presented in Section 4.2 and conduct several experiments. In order to compare the computational experiments implemented for gateway capacity expansion with the gateway network design problem, we start with modifying the capacity, container capacity and number of available container parameters in a way that the total available capacity is equal with all problem instances as presented in Table 4.2.

The computational results are given in the following Table 5.1. `Total_Cap` represents the total available consolidation capacity for each instance. OFV[GD] stands for the objective value of gateway network design problem, whereas OFV[CER] stands for the solutions obtained by relaxation heuristic for the gateway capacity expansion problem. Ratio[GD vs CER] refers to ratio between the objective function values of GD and CER. We might think of the Ratio[GD vs CER] column as representing how much we can improve the gateway capacity expansion problem, if we distribute the initial consolidation capacities efficiently. Time[GD] and Time[CER] refer to the required computational time (in seconds) to solve corresponding instances.

`Total_Cap`	K	OFV[GD]	OFV[CER]	Ratio[GD vs CER]	Time[GD]	Time[CER]
2920000	1000	515366	196762	38.2%	62.9	131.7
3820000		672864	378356	56.2%	55.1	66.4
2920000	10000	513627	196762	38.3%	83.5	131.7
3820000		671126	378356	56.4%	78.9	66.8
8560000	1000	1491020	382449	25.7%	51.5	109.7
9460000		1641010	582619	35.5%	51.2	66.3
8560000	10000	1480890	382449	25.8%	51.9	109.2
9460000		1630910	582619	35.7%	52.6	67.5
14600000	1000	2430840	522617	21.5%	52.4	109.8
15500000		2647670	735650	27.8%	50.9	69.1
14600000	10000	2420970	522617	21.6%	49.4	108.8
15500000		2637550	735650	27.9%	48.8	67.9

Table 5.1: Results for the gateway network design problem vs corresponding gateway capacity expansion problem

According to the Table 5.1, we observe that OFV[GD] reduces as container capacity increases for the equal amount of total available consolidation capacity as the problem becomes more restricted with increasing container capacity provided that the total capacity remains the same.

Due to the pre-determined consolidation capacity for the gateway capacity expansion problem, we are not allowed to utilize selected gateway capacities subject to the total amount of container capacity. This causes a significant loss in profits from consolidation operations. Hence, we observe that the performance gap between GD and CER gets wider as the total available container capacity decreases. Solving the gateway network design problem is computationally less expensive.

Chapter 6

CONCLUSION

We study freight consolidation and routing plans of an air cargo carrier aiming to maximize the savings due to less handling effort at capacitated gateways. The operational problem constructs network-wide consolidation and routing plans given the capacities of gateways and the flight network information. The tactical level problem integrates the capacity expansion decision process based on initial capacity levels. The gateway network design problem determines the hub locations and how the available container capacity would be allocated to those selected hubs without initial capacities.

To solve the freight consolidation and routing problem, we develop a set-covering type linear programming problem formulation where a decision variable represent the routing and consolidation scheme of an OD pair. We propose a column generation algorithm due to computational complexity of enumerating all possible decision variables. According to our computational study, we conclude that column generation approach is almost 9 times faster than solving linear programming problem with a commercial solver.

We extend the set-covering type formulation for the gateway capacity expansion problem; the problem turns out to be a $MILP$ problem. We develop an optimization-based heuristic, which solves the capacity expansion problem in an iterative manner, utilizing the column generation approach as a sub-routine. Different column selection criteria are also studied for the performance of pricing sub-problem process; we conclude that selecting a subset of promising columns entering in the restricted master problem is superior to adding all possible promising columns at each pricing iteration. However, the Relaxation heuristic is the best approach to solve the capacity expansion problem.

Finally, we study the gateway network design problem to determine the hub locations over the flight network. For this purpose, we further extend the set-covering type $MILP$ formulation. We develop a column generation-based algorithm to solve the gateway network design problem iteratively. We investigate the computational results by comparing the gateway network design and capacity expansion problems using variations of the real-life problem.

In air cargo transportation literature, air cargo scheduling and routing and strategical level hub location problems have already been studied individually. However, our study is the first to focus on both cargo consolidation and hub network design problems simultaneously. We also contribute to the literature by focusing on tactical level capacity expansion problem. Our major assumption in this work is the aggregation of time dimension considering that the demand and the availability of resources during a finite planning horizon can be aggregated. We also want to avoid this assumption and consider the demand, the flight network information and the availability of resources over the planning horizon. This is particularly essential for the operational level plans. However, adding the time component will critically change the problem formulations, and therefore the solution procedures.

As the future work, we intend to investigate the performance of a dual ascent procedure for the hub network design problem.

Bibliography

Ahuja, R. K., Magnanti, T. L., and Orlin, J. B. (1993). *Network flows: theory, algorithms, and applications*. Prentice hall.

Amaruchkul, K. and Lorchirachoonkul, V. (2011). Air-cargo capacity allocation for multiple freight forwarders. *Transportation Research Part E: Logistics and Transportation Review*, 47(1):30 – 40.

Barnhart, C., Johnson, E. L., Nemhauser, G. L., Savelsbergh, M. W. P., and Vance, P. H. (1996). Branch-and-price: Column generation for solving huge integer programs. *Operations Research*, 46:316–329.

Bazaraa, M. S., Jarvis, J. J., and Sherali, H. D. (2004). *Linear Programming and Network Flows*. Wiley-Interscience.

Dantzig, G. B. and Wolfe, P. (1960). Decomposition principle for linear programs. *Operations Research*, 8(1):pp. 101–111.

Feng, B., Li, Y., and Shen, Z.-J. M. (2015). Air cargo operations: Literature review and comparison with practices. *Transportation Research Part C: Emerging Technologies*, 56:263 – 280.

Hall, R. W. (1987). Consolidation strategy: Inventory, vehicles and terminals. *Journal of Business Logistics*, 8(2):57–73.

Huang, K. and Chi, W. (2007). A lagrangian relaxation based heuristic for the consolidation problem of airfreight forwarders. *Transportation Research Part C: Emerging Technologies*, 15(4):235 – 245. Modeling and Optimization for Transportation Logistics.

Irnich, S. (2000). A multi-depot pickup and delivery problem with a single hub and heterogeneous vehicles. *European Journal of Operational Research*, 122(2):310 – 328.

Lee, C., Huang, H. C., Liu, B., and Xu, Z. (2006). Development of timed colour petri net simulation models for air cargo terminal operations. *Computers Industrial Engineering*, 51(1):102 – 110.

Li, Y., Tao, Y., and Wang, F. (2009). A compromised large-scale neighborhood search heuristic for capacitated air cargo loading planning. *European Journal of Operational Research*, 199(2):553 – 560.

Li, Z., Bookbinder, J. H., and Elhedhli, S. (2012). Optimal shipment decisions for an airfreight forwarder: Formulation and solution methods. *Transportation Research Part C: Emerging Technologies*, 21(1):17 – 30.

Liu, J., Ahuja, R. K., and Şahin, G. (2008). Optimal network configuration and capacity expansion of railroads. *The Journal of the Operational Research Society*, 59(7):pp. 911–920.

Pan, S., Ballot, E., and Fontane, F. (2013). The reduction of greenhouse gas emissions from freight transport by pooling supply chains. *International Journal of Production Economics*, 143(1):86 – 94.

Pearce, B. (2015). Economic performance of the airline industry. Technical report, IATA.

Schrijver, A. (2003). *Combinatorial optimization : polyhedra and efficiency. Vol. A. , Paths, flows, matchings. chapters 1-38.* Algorithms and combinatorics. Springer-Verlag, Berlin, Heidelberg, New York, N.Y., et al.

Shi, J., Zhang, G., and Sha, J. (2012). A lagrangian based solution algorithm for a build-to-order supply chain network design problem. *Advances in Engineering Software*, 49:21 – 28.

Tyan, J. C., Wang, F.-K., and Du, T. C. (2003). An evaluation of freight consolidation policies in global third party logistics. *Omega*, 31(1):55 – 62.

Xu, T.-L., Luan, K., and Jia, H.-F. (2006). Consolidation strategy and distribution cost of joint distribution. *Chang'an Daxue Xuebao (Ziran Kexue Ban)/Journal of Chang'an University (Natural Science Edition)*, 26(3):68–71.

Zhou, J. and Mueller, M. (2003). Depth-first discovery algorithm for incremental topological sorting of directed acyclic graphs. *Information Processing Letters*, 88(4):195 – 200.